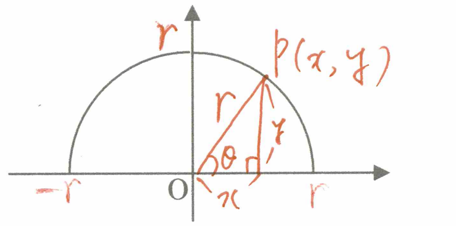
第 21 回　三角比の拡張

三角比の拡張

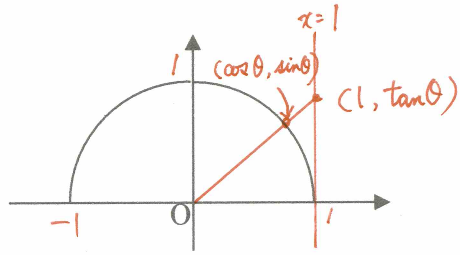
①$0° \leqq \theta \leqq 180°$ の三角比

$\sin\theta = \dfrac{y}{r}$　, $\cos\theta = \dfrac{x}{r}$　, $\tan\theta = \dfrac{y}{x}$



②単位円と三角比

$\sin\theta = y$　, $\cos\theta = x$　, $\tan\theta = \dfrac{y}{x}$



	0°	30°	45°	60°	90°	120°	135°	150°	180°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	なし	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0

Pattern. 1 鈍角の三角比

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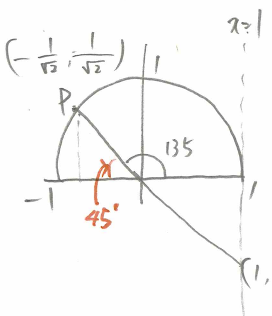
単位円をかいて、座標から求める

（例題 1）次の角度について正弦、余弦、正接の値を求めよ。

(1)135°



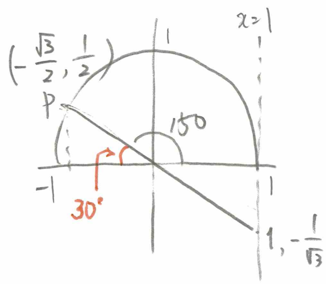
図より

$\sin 135° = \dfrac{1}{\sqrt{2}}$

$\cos 135° = -\dfrac{1}{\sqrt{2}}$

$\tan 135° = -1$

(2)150°



図より

$\sin 150° = \dfrac{1}{2}$

$\cos 150° = -\dfrac{\sqrt{3}}{2}$

$\tan 150° = -\dfrac{1}{\sqrt{3}}$

余角・補角の三角比

①90° $-\theta$ の三角比

$\sin(90^\circ - \theta) =$ $\cos\theta$, $\cos(90^\circ - \theta) =$ $\sin\theta$, $\tan(90^\circ - \theta) =$ $\frac{1}{\tan}$

②180° $-\theta$ の三角比

$\sin(180^\circ - \theta) =$ $\sin\theta$, $\cos(180^\circ - \theta) =$ $-\cos\theta$, $\tan(180^\circ - \theta) =$ $-\tan\theta$

③90° $+\theta$ の三角比

$\sin(90^\circ + \theta) =$ $\cos\theta$, $\cos(90^\circ + \theta) =$ $-\sin\theta$, $\tan(90^\circ + \theta) =$ $-\frac{1}{\tan\theta}$

Pattern. 2 余角・補角の三角比

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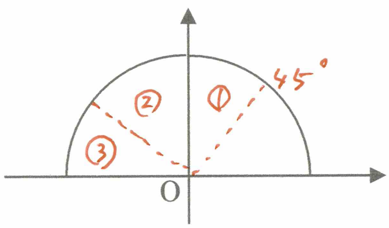
全ての角を45°以下の三角比で書き表す

① $90^\circ - \theta$ ⇒ x, y が逆

② $180^\circ - \theta$ ⇒ cos, tan がマイナス

③ $90^\circ + \theta$ ⇒ ① + ②



(例題 2)次の式を簡単にせよ。

(1) $\cos 170^\circ + \cos 70^\circ + \sin 80^\circ - \sin 160^\circ$

(与式) $= \cos(180^\circ - 10) + \cos(90^\circ - 20^\circ) + \sin(90^\circ - 10^\circ) - \sin(180^\circ - 20)$

$= -\cos 10^\circ + \sin 20^\circ + \cos 10^\circ - \sin 20^\circ$

$= 0$

(2) $\tan 145^\circ \tan 55^\circ - 3\tan 155^\circ \tan 65^\circ$

(与式) $= \tan(180^\circ - 35^\circ) \cdot \tan(90^\circ - 35^\circ) - 3\tan(180^\circ - 25^\circ) \cdot \tan(90^\circ - 25^\circ)$

$= -\tan 35^\circ \cdot \frac{1}{\tan 35^\circ} - 3 \cdot (-\tan 25^\circ) \cdot \frac{1}{\tan 25^\circ}$

$= -1 + 3$

$= 2$