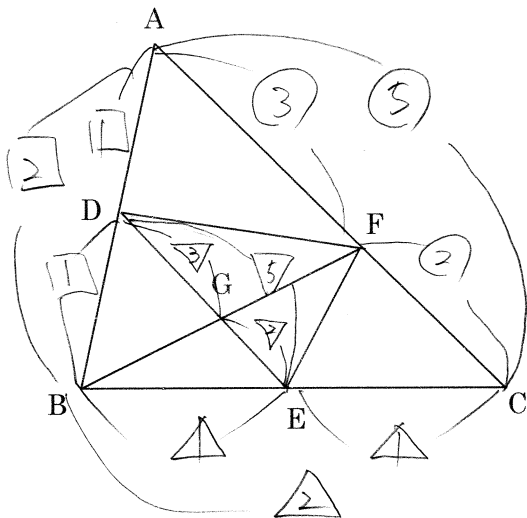


(練習)

右の $\triangle ABC$ において、点D、Eはそれぞれ辺AB、BCの中点です。点Fは辺AC上を $AF:FC=3:2$ に分ける点です。次の問いに答えなさい。



(1) $\triangle ADF : \triangle ABC$ を求めよ。

$$\begin{aligned}\triangle ADF &= \frac{1}{2} \triangle ABF \\ &= \frac{1}{2} \left(\frac{3}{5} \triangle ABC \right) \\ &= \frac{3}{10} \triangle ABC\end{aligned}$$

$$\triangle ADF : \triangle ABC = \underline{3 : 10}$$

(2) $\triangle BDF : \triangle BEF$ を求めよ。

$$\begin{aligned}\triangle BDF &= \frac{1}{2} \triangle ABF & \triangle BEF &= \frac{1}{2} \triangle BCF \\ &= \frac{1}{2} \left(\frac{3}{5} \triangle ABC \right) & &= \frac{1}{2} \left(\frac{2}{5} \triangle ABC \right) \\ &= \frac{3}{10} \triangle ABC & &= \frac{1}{5} \triangle ABC\end{aligned}$$

$$\begin{aligned}\triangle BDF : \triangle BEF &= \frac{3}{10} : \frac{1}{5} \\ &= \underline{3 : 2}\end{aligned}$$

(3) $DG : GE$ を求めよ。

$$\begin{aligned}DG : GE &= \triangle BDF : \triangle BEF \leftarrow \text{底辺BF共通} \\ &= \underline{3 : 2}\end{aligned}$$

(4) $\triangle DFG$ は $\triangle ABC$ の何倍ですか。

$$\begin{aligned}\triangle DFG &= \frac{3}{5} \triangle DEF \\ &= \frac{3}{5} \left(\frac{1}{4} \triangle ABC \right) \\ &= \frac{3}{20} \triangle ABC\end{aligned}$$

$$\begin{aligned}\triangle BDE &= \frac{1}{2} \triangle ABE \\ &= \frac{1}{2} \left(\frac{1}{2} \triangle ABC \right) \\ &= \frac{1}{4} \triangle ABC\end{aligned}$$

$$\begin{aligned}\triangle CEF &= \frac{1}{2} \triangle BCF \\ &= \frac{1}{2} \left(\frac{2}{5} \triangle ABC \right) \\ &= \frac{1}{5} \triangle ABC\end{aligned}$$

$$\begin{aligned}\triangle DEF &= \triangle ABC - (\triangle ADF + \triangle BDF + \triangle CEF) \\ &= \left\{ 1 - \left(\frac{3}{10} + \frac{1}{4} + \frac{1}{5} \right) \right\} \triangle ABC \\ &= \left(\frac{20}{20} - \frac{15}{20} \right) \triangle ABC \\ &= \frac{1}{4} \triangle ABC\end{aligned}$$

(発展1)

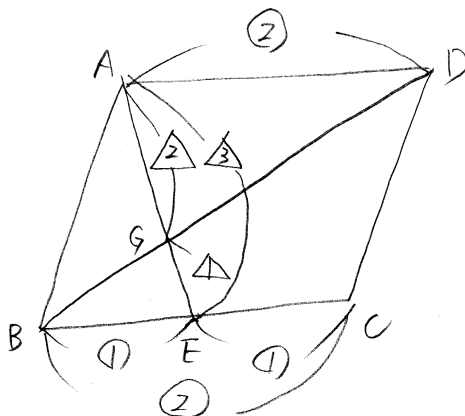
$$(1) \triangle BEG = \frac{1}{3} \triangle ABE$$

$$= \frac{1}{3} \left(\frac{1}{2} \triangle ABC \right)$$

$$= \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} \square ABCD$$

$$= \frac{1}{12} \square ABCD$$

$$\triangle BEG : \square ABCD = 1 : 12$$



$$\left(\triangle BEG \text{ is } \triangle DAG \text{ of } \frac{1}{2} \text{ of the rectangle } \therefore \text{EG} : \text{AG} = 2 : 1 \right)$$

$$(2) \triangle AEF : \square ABCD$$

$$\triangle ABE = \frac{1}{2} \triangle ABC$$

$$= \frac{1}{2} \left(\frac{1}{2} \square ABCD \right)$$

$$= \frac{1}{4} \square ABCD$$

$$\triangle CEF = \frac{1}{2} \triangle BCF$$

$$= \frac{1}{2} \left(\frac{1}{2} \triangle BCD \right)$$

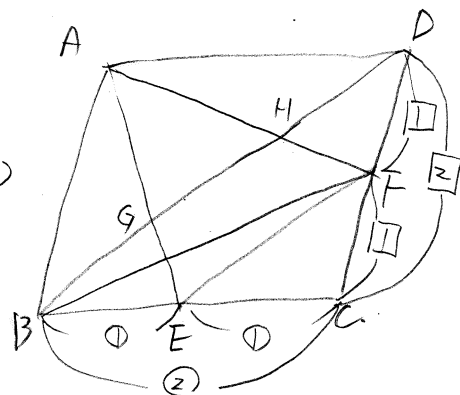
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \square ABCD$$

$$= \frac{1}{8} \square ABCD$$

$$\triangle ADF = \frac{1}{2} \triangle ACD$$

$$= \frac{1}{2} \left(\frac{1}{2} \square ABCD \right)$$

$$= \frac{1}{4} \square ABCD$$



$$\triangle AEF = \square ABCD - (\triangle ABE + \triangle ADF + \triangle CEF)$$

$$= \left\{ 1 - \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{8} \right) \right\} \square ABCD$$

$$= \left(1 - \frac{5}{8} \right) \square ABCD$$

$$= \frac{3}{8} \square ABCD$$

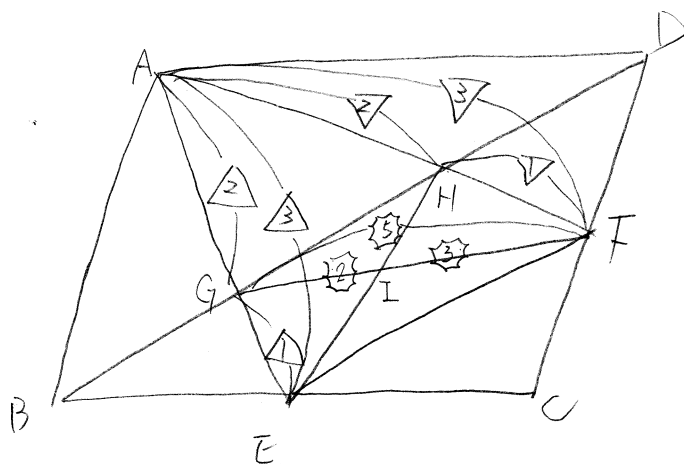
$$\triangle AEF : \square ABCD$$

$$= 3 : 8$$

$$(3) \triangle AGH : \triangle AEF$$

$$\begin{aligned} \triangle AGH &= \frac{2}{3} \triangle AEH \\ &= \frac{2}{3} \left(\frac{2}{3} \triangle AEF \right) \\ &= \frac{4}{9} \triangle AEF \end{aligned}$$

$$\left[\begin{aligned} &= \frac{4}{9} \left(\frac{3}{8} \square ABCD \right) \\ &= \frac{1}{6} \square ABCD \end{aligned} \right]$$



$$\left(\begin{aligned} &\triangle DFH \text{ は } \triangle ABH \text{ の } \frac{1}{2} \text{ の縮図だから} \\ &AH : HF = 2 : 1 \end{aligned} \right)$$

$$\triangle AGH : \triangle AEF$$

$$= \underline{4 : 9}$$

$$(4) \triangle EGH : \triangle EFH$$

$$= \frac{2}{9} : \frac{1}{3}$$

$$= \underline{2 : 3}$$

$$\triangle EGH = \frac{1}{3} \triangle AEH \quad \triangle EFH = \frac{1}{3} \triangle AEF$$

$$= \frac{1}{3} \left(\frac{2}{3} \triangle AEF \right)$$

$$= \frac{2}{9} \triangle AEF$$

$$(5) \triangle GHJ : \triangle EFI$$

$$= \frac{1}{30} : \frac{3}{40}$$

$$= 40 : 90$$

$$= \underline{4 : 9}$$

$$\triangle GHJ = \frac{2}{5} \triangle GFH$$

$$= \frac{2}{5} \left(\frac{1}{3} \triangle AGF \right)$$

$$= \frac{2}{5} \times \frac{1}{3} \left(\frac{2}{3} \triangle AEF \right)$$

$$= \frac{2}{5} \times \frac{1}{3} \times \frac{2}{3} \times \frac{3}{8} \square ABCD$$

$$= \frac{1}{30} \square ABCD$$

$$\triangle EFI = \frac{3}{5} \triangle EFH$$

$$= \frac{3}{5} \left(\frac{1}{3} \triangle AEF \right)$$

$$= \frac{3}{5} \times \frac{1}{3} \left(\frac{3}{8} \square ABCD \right)$$

$$= \frac{3}{40} \square ABCD$$

(3)(5) の結果が同じなのって

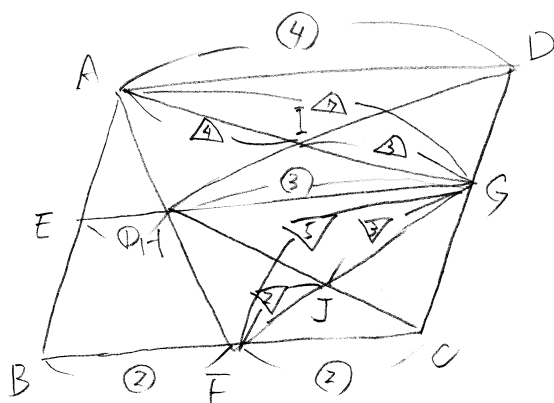
偶然ではなく、必然。拡大図・縮図では図形の面積比は縮尺の比の2乗に比例する。

$$\text{縮尺 } 2 : 3 \rightarrow \text{面積比 } 2^2 : 3^2$$

$$= \underline{4 : 9}$$

(発展2)

$$\begin{aligned}
 (1) \triangle AFG &= \square ABCD - (\triangle ABF + \triangle ADG + \triangle CFG) \\
 &= \left\{ 1 - \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{2} \right) \right\} \square ABCD \\
 &= \left(1 - \frac{5}{8} \right) \square ABCD \\
 &= \frac{3}{8} \square ABCD
 \end{aligned}$$



$$\begin{aligned}
 (2) \triangle AID &= \frac{3}{7} \triangle ADG \\
 &= \frac{3}{7} \left(\frac{1}{2} \triangle ACD \right) \\
 &= \frac{3}{7} \times \frac{1}{2} \left(\frac{1}{2} \square \right) \\
 &= \frac{3}{28} \square
 \end{aligned}$$

$$\begin{aligned}
 \triangle JFC &= \frac{2}{5} \triangle CFG \\
 &= \frac{2}{5} \left(\frac{1}{2} \triangle BCG \right) \\
 &= \frac{2}{5} \times \frac{1}{2} \left(\frac{1}{2} \triangle BCD \right) \\
 &= \frac{2}{5} \times \frac{1}{2} \times \frac{1}{2} \left(\frac{1}{2} \square \right) \\
 &= \frac{1}{20} \square
 \end{aligned}$$

$$\begin{aligned}
 \triangle ABF &= \frac{1}{2} \triangle ABC & \triangle ADG &= \frac{1}{2} \triangle ADC \\
 &= \frac{1}{2} \left(\frac{1}{2} \square \right) & &= \frac{1}{2} \left(\frac{1}{2} \square ABC \right) \\
 &= \frac{1}{4} \square & &= \frac{1}{4} \square
 \end{aligned}$$

$$\begin{aligned}
 \triangle CFG &= \frac{1}{2} \triangle BCG \\
 &= \frac{1}{2} \left(\frac{1}{2} \triangle BCD \right) \\
 &= \frac{1}{2} \times \frac{1}{2} \left(\frac{1}{2} \square \right) \\
 &= \frac{1}{8} \square
 \end{aligned}$$

$$\begin{aligned}
 \triangle AID &: \triangle JFC \\
 &= \frac{3}{28} : \frac{1}{20} \\
 &= 20 : 28 \\
 &= 60 : 28 \\
 &= 15 : 7
 \end{aligned}$$

A. $\frac{15}{7}$ 倍

$$\begin{aligned}
 (3) \triangle GHI &= \frac{3}{7} \triangle AGH \\
 &= \frac{3}{7} \left(\frac{1}{2} \triangle AFG \right) \\
 &= \frac{3}{7} \times \frac{1}{2} \left(\frac{3}{8} \square \right) \\
 &= \frac{9}{112} \square
 \end{aligned}$$

$$\begin{aligned}
 \triangle GHJ &= \frac{3}{5} \triangle GFH \\
 &= \frac{3}{5} \left(\frac{1}{2} \triangle AFG \right) \\
 &= \frac{3}{5} \times \frac{1}{2} \left(\frac{3}{8} \square \right) \\
 &= \frac{9}{80} \square
 \end{aligned}$$

$$\begin{aligned}
 \text{四角形IHJG} &= \triangle GHI + \triangle GHJ \\
 &= \left(\frac{9}{112} + \frac{9}{80} \right) \square \\
 &= \frac{108}{560} \square \\
 &= \frac{27}{140} \square
 \end{aligned}$$

A. $\frac{27}{140}$ 倍