

(7) 平方根の四則混合計算

(例1) 四則混合計算…最初と最後に、分母の有理化と $a\sqrt{b}$ の形にするのを忘れない。

$$\begin{aligned} \textcircled{1} \sqrt{18} - \sqrt{24} \times \sqrt{12} &= 3\sqrt{2} - \sqrt{2 \times 12 \times 12} \\ &= 3\sqrt{2} - 2\sqrt{6} \times 2\sqrt{3} \\ &= 3\sqrt{2} - 4\sqrt{18} \\ &= 3\sqrt{2} - 12\sqrt{2} \\ &= -9\sqrt{2} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \sqrt{3}(\sqrt{2} - \sqrt{15}) - \frac{\sqrt{8}}{\sqrt{3}} \times \sqrt{3} &= \sqrt{6} - \sqrt{45} - \frac{2\sqrt{6}}{3} \\ &= \frac{3\sqrt{6}}{3} - 3\sqrt{5} - \frac{2\sqrt{6}}{3} \\ &= \frac{\sqrt{6}}{3} - 3\sqrt{5} \end{aligned}$$

(例2) 乗法公式の利用… $(a\sqrt{b})^2 = a^2 b$ に注意!

$$\begin{aligned} \textcircled{1} (\sqrt{2} + 2)(\sqrt{2} - 3) &= (\sqrt{2})^2 + (2-3)\sqrt{2} + 2 \times (-3) \\ &= 2 - \sqrt{2} - 6 \\ &= -4 - \sqrt{2} \end{aligned}$$

$$\begin{aligned} \textcircled{2} (2\sqrt{2} - \sqrt{3})^2 &= (2\sqrt{2})^2 + 2 \times 2\sqrt{2} \times (-\sqrt{3}) + (-\sqrt{3})^2 \\ &= 8 - 4\sqrt{6} + 3 \\ &= 11 - 4\sqrt{6} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \frac{(3\sqrt{2} - \sqrt{24})^2}{2\sqrt{8}} - \frac{(\sqrt{18} + 2\sqrt{6})^2}{3\sqrt{2}} &= \frac{18 - 12\sqrt{12} + 24}{2\sqrt{8}} - \frac{(18 + 12\sqrt{12} + 24)}{3\sqrt{2}} \\ &= \frac{18 - 24\sqrt{3} + 24}{2\sqrt{8}} - \frac{18 + 24\sqrt{3} + 24}{3\sqrt{2}} \\ &= \frac{-48\sqrt{3}}{2\sqrt{8}} \end{aligned}$$

$$\begin{aligned} * \textcircled{4} \frac{1}{\sqrt{3} - \sqrt{2}} - \frac{\sqrt{2}}{2} &= \frac{1}{\sqrt{3} - \sqrt{2}} \times \frac{(\sqrt{3} + \sqrt{2})}{(\sqrt{3} + \sqrt{2})} - \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{3} + \sqrt{2}}{3 - 2} - \frac{\sqrt{2}}{2} \\ &= \sqrt{3} + \sqrt{2} - \frac{\sqrt{2}}{2} \\ &= \sqrt{3} + \frac{2\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \\ &= \sqrt{3} + \frac{\sqrt{2}}{2} \end{aligned}$$

$$= \sqrt{3} + \frac{\sqrt{2}}{2} \left(\frac{2\sqrt{2} + \sqrt{2}}{2} \right)$$

$$\begin{aligned} \textcircled{4} a^2 - b^2, (a+b)(a-b) \text{ の利用} & \\ (3\sqrt{2} - 2\sqrt{6} + 3\sqrt{2} + 2\sqrt{6}) \times \left\{ \begin{array}{l} 3\sqrt{2} - 2\sqrt{6} \\ \ominus (3\sqrt{2} + 2\sqrt{6}) \\ \hline -3\sqrt{2} - 2\sqrt{6} \end{array} \right\} & \\ = 6\sqrt{2} \times (-4\sqrt{6}) & \\ = -24\sqrt{12} & \\ = -48\sqrt{3} & \end{aligned}$$

(8) 式の値

(例3) ① $x = \sqrt{5} + 2$, $y = \sqrt{5} - 2$ のとき ② $x + y = \sqrt{3} + 1$, $xy = \sqrt{3} - 1$ のとき
 $x^2 - xy$ の値を求めよ。 $x^2 - xy + y^2$ の値を求めよ。

$$\begin{aligned}x^2 - xy &= x(x - y) \leftarrow \text{代入} \\ &= (\sqrt{5} + 2) \left\{ \begin{array}{l} \sqrt{5} + 2 - (\sqrt{5} - 2) \\ - \sqrt{5} + 2 \end{array} \right\} \\ &= (\sqrt{5} + 2) \times 4 \\ &= \underline{4\sqrt{5} + 8}\end{aligned}$$

$$\begin{aligned}& \frac{x^2 + y^2}{\quad} - xy \\ &= \frac{(x + y)^2 - 2xy}{\quad} - xy \\ &= (x + y)^2 - 3xy \leftarrow \text{代入} \\ &= (\sqrt{3} + 1)^2 - 3(\sqrt{3} - 1) \\ &= 3 + 2\sqrt{3} + 1 - 3\sqrt{3} + 3 \\ &= \underline{7 - \sqrt{3}}\end{aligned}$$

(9) 平方根の利用

(例4) ① $\sqrt{24n}$ が整数となるような自然数 n を小さい方から3つかけ。

$$\begin{aligned}24 &= 2^3 \times 3 \\ &= 2^2 \times 2^0 \times 3^0 \quad \leftarrow 2 \text{乗する区切る}\end{aligned}$$

$$\begin{cases} n_1 = 2 \times 3 = 6 \\ n_2 = 6 \times 2^2 = 24 \\ n_3 = 6 \times 3^2 = 54 \end{cases}$$

↑ 順番に2乗をかける

A. $n = 6, 24, 54$

② $\sqrt{\frac{300}{n}}$ が整数となるような自然数 n を全て求めよ。

$$300 = 2^2 \times 3^0 \times 5^2 \quad \sqrt{\frac{2^2 \times 3 \times 5^2}{n}}$$

$$\begin{cases} n_1 = 3 \\ n_2 = 3 \times 2^2 = 12 \\ n_3 = 3 \times 5^2 = 75 \\ n_4 = 3 \times 2^2 \times 5^2 = 300 \end{cases}$$

A. $n = 3, 12, 75, 300$

↑ 分子にある2乗を順番に!

(裏に続く)

③ $\sqrt{50-2n}$ が整数となるような自然数 n を全て求めよ。

50より小さい2乗の数 $\rightarrow 49, 36, 25, 16, 9, 4, 1, 0$ 注意! (偶数だけ)
 (考え方は5n)

$$\begin{aligned} 50-2n &= 36 \\ 2n &= 14 \\ n &= 7 \end{aligned}$$

$$\begin{aligned} 50-2n &= 16 \\ 2n &= 34 \\ n &= 17 \end{aligned}$$

$$\begin{aligned} 50-2n &= 4 \\ 2n &= 46 \\ n &= 23 \end{aligned}$$

$$\begin{aligned} 50-2n &= 0 \\ 2n &= 50 \\ n &= 25 \end{aligned}$$

A $n = 7, 17, 23, 25$

(例5) $\sqrt{7}$ の整数部分を a , 小数部分を b とするとき、 $\sqrt{28a-4b}$ の値を求めよ。

☆ \sqrt{a} の小数部分 = $\sqrt{a} - (\sqrt{a}$ の整数部分)

$4 < 7 < 9$
 $2 < \sqrt{7} < 3$ \leftarrow 平方根

$$\begin{aligned} \sqrt{28a-4b} &= 2\sqrt{7} \times 2(\sqrt{7}-2) \\ &= 4\sqrt{7} - 4\sqrt{7} + 8 \\ &= 8 \end{aligned}$$

$a = 2$
 $b = \sqrt{7} - 2$

<乗法公式を使った分母の有理化> ☆ $(a+b)(a-b) = a^2 - b^2$ を利用する

(例6) 次の数の分母を有理化しなさい。

$$\begin{aligned} \textcircled{1} \frac{1}{\sqrt{3}-\sqrt{2}} &\times \frac{(\sqrt{3}+\sqrt{2})}{(\sqrt{3}+\sqrt{2})} \\ &= \frac{\sqrt{3}+\sqrt{2}}{3-2} \\ &= \underline{\underline{\sqrt{3}+\sqrt{2}}} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \frac{2\sqrt{2}}{3-\sqrt{2}} &\times \frac{(3+\sqrt{2})}{(3+\sqrt{2})} \\ &= \frac{6\sqrt{2}+2 \times 2}{9-2} \\ &= \underline{\underline{\frac{6\sqrt{2}+4}{7}}} \end{aligned}$$